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Reflection Charts Relating to Impedance Matching

HAROLD A. WHEELER, LIFE FELLOW, IEEE

Abstract —**A reflection chart is some grid of coordinates on which to plot an impedance locus over a frequency range. Taking as a reference a constant real impedance, one may construct contours of the reflection coefficient (or the related VSWR, reflection loss, etc.). The reference may be the wave impedance of a transmission line. This may be a line connecting radio equipment with an antenna, or it may be a standard line used in measuring the impedance. The reflection chart in widest use is the so-called "Smith Chart" proposed by Philip H. Smith in 1939. It is one form of the hemisphere chart, which was proposed, also in 1939, by Philip S. Carter. Its properties and uses are described. It has some limitations. A reference value must be assigned, after which the shape of a locus depends on this value. Also, a locus is crowded toward the rim of the chart. A logarithmic reflection chart has recently been proposed by the author, which overcomes these limitations but loses some desirable features of the hemisphere chart.**

I. Introduction

A REFLECTION CHART is a pair of coordinates on which to plot an impedance locus over a range of frequency. The complex impedance may be described in rectangular or polar coordinates. The impedance may be expressed by a ratio over a reference value (Z_0), which is customarily the constant real wave impedance of a transmission line or cable. Then this ratio determines the reflection loss in the transfer of power between a device having the general impedance and a device having the reference impedance.

The most widely known of reflection charts is the so-called Smith Chart, which was first published 45 years ago in 1939 [6]. It is one form of the hemisphere chart. On a circular area, there is an orthogonal grid of circular lines marked with the real and imaginary components of the impedance ratio. These cover the entire range of impedance with positive-real part. This feature is peculiar to any hemisphere chart.

There are various uses of the hemisphere cart. Smith emphasized its utility for computations with the aid of a radial scale pivoted at the center of the chart. Typical computations were series and parallel impedance, and the transformation of impedance through a section of line. The radial scale could be calibrated in any function of the reflection coefficient (ρ), such as the reflection loss at a junction or the voltage standing-wave ratio (VSWR $= S$) in a line terminated in the impedance. Carter, in his simultaneous publication [5], emphasized the use of the hemisphere chart with a standing-wave indicator to measure the impedance ratio of a load on a line. On the circular area, he showed a grid of circular lines marked with the magnitude and angle of the impedance ratio, corresponding to latitude and longitude on a hemisphere. The most advanced equipment for impedance measurement at high radio frequencies (say above 1 MHz) uses an automatic mechanical plotter on the Smith Chart, with an option of digital readout of the reflection coefficient (magnitude and angle) [25].

The hemisphere chart, by virtue of its orthogonal circular coordinates, offers much opportunity for displaying the frequency behavior of an impedance network and various relations, such as resonance. One common application is the wide-band matching of a load that has some limitation on its bandwidth, such as a resonant antenna.

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One should bear in mind that the reflection coefficient is essentially the output of a bridge which is balanced for the reference value of impedance. The association of the measurement with a transmission line enables one to picture the significance of the output. Also, it enables the use of a length of line between the unknown impedance and the measuring device. The hemisphere chart best displays the operation of such a system.

The hemisphere chart requires a choice of the reference value. Then it offers the clearest display of impedance variation in the vicinity of this value. Far from this value, the display is compressed near the rim of the chart.

There are some other sets of coordinates that offer some different opportunities for the display of an impedance locus and its implied reflection coefficient against a reference value. The following were used by the author for plotting an impedance locus that would have the same size and shape at any impedance level. It is implicit that the same would be true of a locus of constant reflection, but it would not be a circle.

- One is the log of the complex impedance [15]. It has parallel boundaries at one quadrant of angle, with compression and distortion of the locus near either extreme of angle.
- Another is the "logarithmic reflection chart" [21], [23]. Its log–log coordinates enable a display of impedance magnitude and angle with no boundaries and, hence, no compression. It gives equal weight to reflection by magnitude ratio or impedance angle.

This account is intended to place in perspective the various concepts and practical applications of the reflection chart as a grid on which to plot an impedance locus over a range of frequency.

II. Symbols

The letter symbols used herein are those I chose in 1948 [14]. With few exceptions, they correspond to P.H. Smith's book published in 1969 [19].

The planes of complex impedance ratio and reflection ratio are described for separate identification and interrelations, as follows:

$$z = Z/Z_0 = z \exp j\phi = r + jx = R/Z_0 + jX/Z_0$$

$$w = -w \exp - j2\beta = u + jv = \frac{z-1}{z+1}$$

$$w = \rho = \frac{S-1}{S+1}$$

$S = r$ on scale of upper vertical radius of hemisphere chart.

The peculiar polar definition of the voltage–reflection ratio w is chosen for several reasons.

- The negative sign of w places the angular origin at the origin of impedance ratio ($z = 0$).
- The angle -2β is the angle of reflection in a line of length β radians or $\beta/2\pi$ wavelengths.
- The short-circuit reactance of a nondissipative line (so $w = 1$) is $x = \tan\beta$.

Both $\beta/2\pi$ and x are to be scaled on the rim of the chart.

z_0 = reference impedance (wave impedance of line) (constant pure resistance) (ohms)

$Z = Z \exp j\phi = R + jX$ = complex impedance (ohms)

$z = Z/Z_0 = z \exp j\phi = r + jx$ = complex impedance ratio

Z, ϕ = polar components of impedance (ohms, radians)

R, X = series rectangular components of impedance (ohms)

$w = -w \exp - j2\beta = u + jv$ = complex voltage–reflection ratio

$w = \rho = \frac{S-1}{S+1}$ = scalar reflection ratio (coefficient) (radius on hemisphere chart)

β = angle length of line (Z_0)

$d = \beta/2\pi$ = length of line in wavelengths

$S = \frac{1+\rho}{1-\rho}$ = voltage standing-wave ratio (VSWR > 1)

$S = r$ on scale of upper radius on vertical axis of hemisphere chart.

See special symbols for logarithmic reflection chart.

III. History

The background of the reflection chart is the plotting of an impedance locus over a frequency range. The coordinates are the two parts of a complex impedance (say R and X). The locus is marked with a frequency scale.

An early example is the motional-impedance circle for a telephone receiver with a resonant diaphragm. It was described in Everitt's textbook in 1932 and 1937 [4]. Over the audio-frequency range, the impedance is the sum of two complex components. One gives a monotonic locus representing the driving coil. It can be measured with the diaphragm blocked. The other gives a superposed circle representing the interaction with the mechanical resonance of the diaphragm.

The use of a slotted line with a sliding probe was proposed for impedance measurements before the advent of the hemisphere chart. Various charts were proposed for computation of the complex impedance from such observations. They lacked the principal attractions of the hemisphere chart.

The evolution of the hemisphere chart occurred in the 1930's. Two milestones are worthy of mention. Each one is a locus of the constant reflection coefficient plotted on rectangular coordinates of complex impedance over the positive-real half-plane. The first was made by Smith at BTL in 1931 but not published until much later [1], [19]. The contours of the constant reflection coefficient appeared as confocal circles. The second was published by the author in 1936 [2], [3]. Similar contours were identified with the constant reflection loss and, hence, with the constant reflection coefficient. The latter was made for the purpose of evaluating wide-band matching networks associated with a resonant antenna [3].

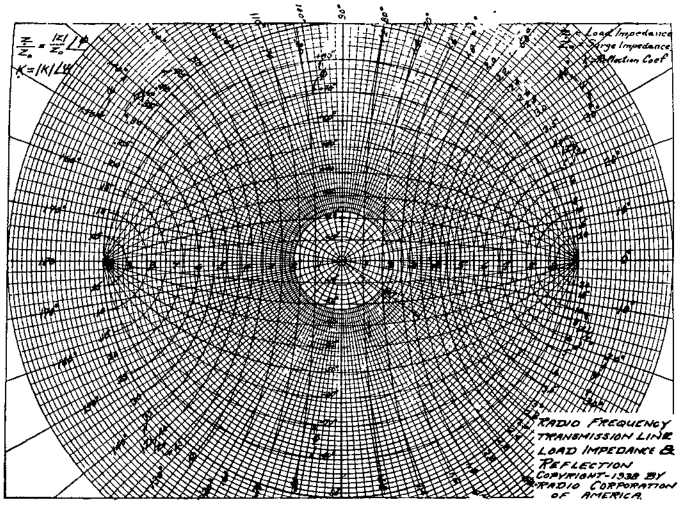


Fig. 1. The Carter Chart (*RCA Review*, 1939).

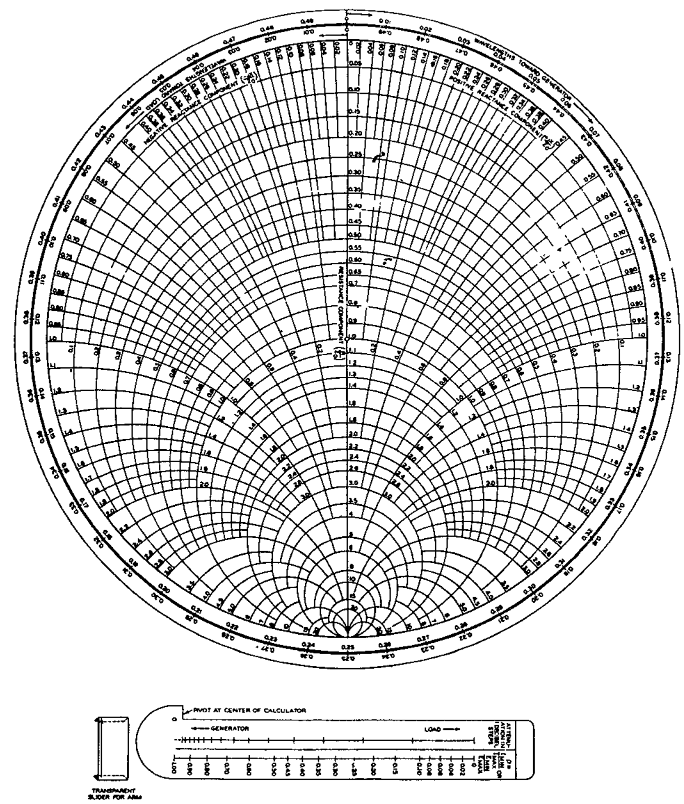


Fig. 2. The Smith Chart (*Electronics*, 1939).

Then, the hemisphere chart was conceived to display the entire range of positive-real impedance on the area of a circle. In the same month (January 1939), the hemisphere reflection chart was published by Philip S. Carter in *RCA Review* [5] and by Philip H. Smith of BTL in *Electronics* [6]. These are reproduced as Figs. 1 and 2. The hemisphere chart as a projection of one-half the impedance sphere was published by E. U. Condon in 1942 [7].

I derived independently the hemisphere chart in a period ending on April 4, 1941. A few days later, Phil Carter was in my office for an IRE committee meeting and I told him about it. He said it was a good idea, and that he had published it two years before.

In the Little Neck Laboratory of Hazeltine Corporation, we were soon using long slotted lines for measuring TV antennas around 40 MHz. Our war work on IFF led to slotted-line measurements of antennas at 150–200 MHz and upward. In 1942, we printed our version of the Carter Chart in quantities for use as graph paper [8]. I prepared numerous memos on its features and its use. On January 14, 1943, I described the hemisphere chart to the Radio Club of America in their regular meeting at Columbia University. Gradually, the Smith Chart came into common use in other laboratories.

We adopted the one logical orientation of the hemisphere chart [8]. All others adopted one or the other of the different orientations of Carter and Smith. I published a comprehensive presentation of our views in 1948 [14].

A major development was automatic plotting on the hemisphere chart. Instantaneous display of a locus on a cathode-ray tube was described by Arthur L. Samuel in 1947 [13]. Now, automatic mechanical plotting is employed in the most advanced measuring equipment for high radio frequencies (say above 1 MHz) [25].

The hemisphere chart has been found to be very useful in presenting and manipulating impedance computations for various purposes. One of the most common is the design of double tuning for wide-band matching between a line and a resonant antenna.

Using the hemisphere chart, we took for granted one peculiarity of that chart. I am referring to the need for assigning a reference value for the center, and the problem of compression of a locus far from the center, approaching the rim of the chart. I came to use some alternatives that overcame this objection, partially or wholly, as mentioned above.

In 1948 [15], I plotted some impedance loci on a rectangular grid of the logarithm of impedance. Its coordinates were magnitude and angle with compatible scales (napiers and radians, or their equivalent decades and degrees). (See also [18].) My immediate motivation was the construction of a ratio mean between two points by bisecting a connecting straight line. On this grid, it was implicit that the size and shape of a locus was independent of magnitude, but there remained the compression near the boundary at either extreme of angle ($\pm 90°$). If used as a reflection chart, it would offer only a half-solution of the compression problem.

Many years later, I sought a real solution to the problem of compression near a boundary. The result was a unique solution, from the viewpoint of a reflection chart. It is the "logarithmic reflection chart," which I published in 1983 [21], [23]. The impedance magnitude and angle are plotted on log–log rectangular coordinates. On the vertical real axis, the log of magnitude corresponds to the log of VSWR. A compatible horizontal scale is derived as the log of VSWR caused by reactance only (with resistance "matched"). On these scales, a reflection locus is centered on the vertical axis and has quadrantal symmetry. Far from the axis, a locus is distorted but does not suffer compression. This chart loses the simplicity of the circular loci on the hemisphere chart but gains some qualities that I have found extremely useful in the synthesis of wide-band impedance-matching networks. It is remarkable that another "unique" reflection chart should have evolved so long (four decades) after the hemisphere chart.

The history of the use of reflection charts has two branches. They may be correlated with the first two publications in 1939. One is the use emphasized by Carter [5], making observations in a transmission line and deducing the impedance of a load on the line. The other is the use emphasized by Smith [6], for graphical computation of the result of adding or subtracting a complex impedance in series or in parallel, and the inversion of impedance.

The measurement application originally relied on standing-wave observations by a sliding probe in a slotted line to determine the reflection coefficient. Subsequently, the directional coupler enabled the direct measurement of the reflection coefficient, amplitude, and phase, from which a point could be automatically plotted on the hemisphere chart. A coordinate grid could be chosen for reading any related set of numbers, such as R and X of impedance Z. This is the most common method of measuring impedance at high radio frequencies (say above 1 MHz).

The computation application is commonly used in conjunction with measurements. It displays the reflection coefficient, perhaps in terms of VSWR tolerance of mismatch over a frequency band. It is convenient for graphical computation of the effect of series or parallel impedance, especially reactors in a matching network. If the computation is performed by a numerical computer, the result may be plotted on the hemisphere chart.

Aside from graphical computation, the reflection chart is useful mainly for displaying a frequency locus of impedance in such a way as to be most meaningful on sight. The logarithmic reflection chart may offer the greatest utility in this function.

IV. Projections of the Impedance Sphere

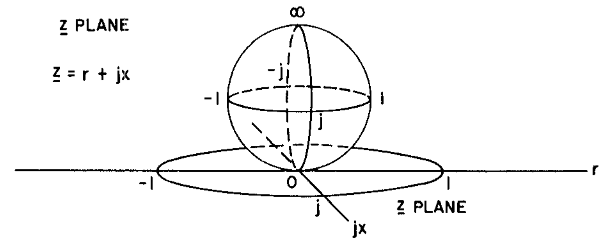
Relative to a constant-real reference value, the complex impedance on a plane can be projected on a sphere [7], [14]. Fig. 3(a) shows this projection. The positive-real half-plane is projected on one hemisphere, which gives the name to the hemisphere chart. The unit impedance ratio is at the center of this hemisphere.

Fig. 3(b) shows the projection of this hemisphere on a plane tangent at the center. The circle bounding this projection is the locus of the unit coefficient of reflection, with zero at the center. This circle becomes the hemisphere chart of the reflection coefficient, on which may be plotted the impedance ratio.

These projections of the impedance sphere were described by Edward U. Condon in 1942 [7], and I should have given him credit in my 1948 monograph [14].

In each of these projections, a circle is projected as a circle. (A straight line is a special case of a circle.) It follows that a circle on either plane is transformed to a circle on the other. For example, the R and X contours are straight lines on the impedance plane and, hence, circles on the reflection plane. This fact is responsible for the simplicity of the circle chart and its versatility in geometric construction of various relations.

(a)



(b)

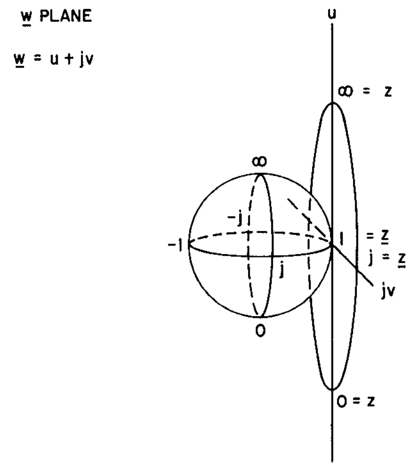


Fig. 3. Projections of the impedance sphere. (a) On the plane of impedance ratio. (b) On the plane of reflection ratio.

V. Orientations of the Hemisphere Charts

The graphical presentation of engineering relationships should be made in a form adopted for logical reasons. A principal consideration is the communication of significant information. Another consideration is consistent use of the logical form.

Coordinates for plotting an impedance locus were first taken for granted from a precedent for a different purpose. The slavish adherence to that convention had led to common use of an orientation that is devoid of logic. I refer to the Carter Chart and to the later form of the Smith Chart. It lies on its side. I immediately adopted an equivalent form but right-side up. The history of these practices is worthy of note. It is relevant to the hemisphere chart and any other grid for plotting a locus of complex impedance.

Fig. 4 shows some options in the choice of a pair of rectangular coordinates. Fig. 4(a) is the classical orientation for plotting $y = f(x)$. It was natural for that purpose and did not offend any considerations of logic.

With the theory of functions of a complex variable, there was a need for plotting a locus of simultaneous variation of two coordinate-dependent variables with some independent variable. An example is the frequency locus of $\boldsymbol{Z} = R(\omega) + jX(\omega)$. Because a complex variable was described as $z = x + jy$, it was mapped on the same coordinates, as

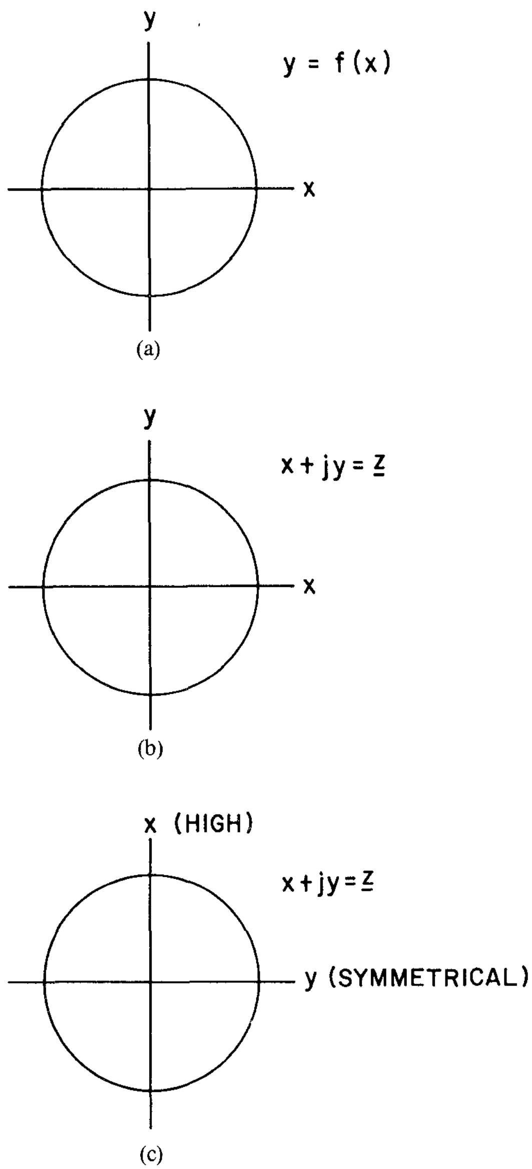


Fig. 4. Choice of rectangular coordinates for two-dimensional plotting on a plane. (a) Classical pair of coordinates for independent and dependent variables. (b) Conventional adoption of same coordinates for map of coordinate parts of complex variable. (c) Logical coordinates for map of complex variable (especially the positive-real half-plane). Smith, 1931 [1]; Wheeler, 1936 [8], [9].

seen in Fig. 4(b). This practice disregarded some logical considerations.

A thoughtful election would have led to the practice shown in Fig. 4(c).

- First, the axis of the imaginary part (y) typically has paired values and symmetry, so it is natural to use a left–right orientation.
- Secondly, the positive-real scale is naturally associated with increasing or "higher" values, so it is natural to use an upward orientation.

Both of these considerations are especially relevant to a map of $\boldsymbol{Z}(\omega)$, with which we are here concerned. It is notable that Smith and I chose this orientation for our first

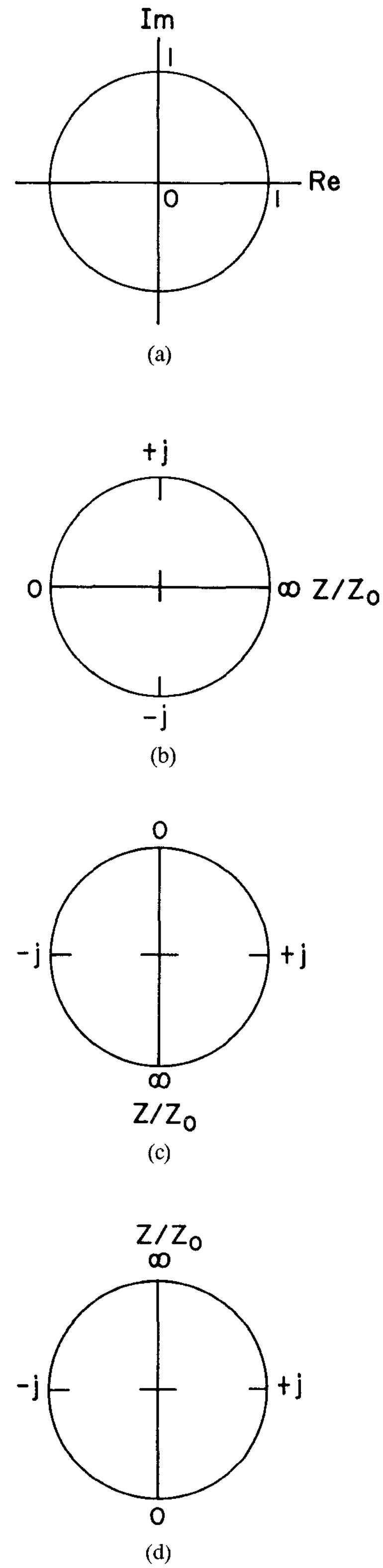


Fig. 5. Choice of hemisphere-chart coordinates for map of positive-real hemisphere of impedance ratio. (a) Conventional coordinates for map of reflection coefficient on hemisphere chart. Carter, 1939 [5]. (b) Conventional display of positive-real hemisphere of impedance ratio. Carter, 1939 [5]; Smith later [19]. (c) Upside-down display of positive-real hemisphere of impedance ratio. Smith, 1939 [6] and 1944 [10]. (d) Logical display of positive-real hemisphere of impedance ratio. Wheeler, 1942 [8] and 1948 [14].

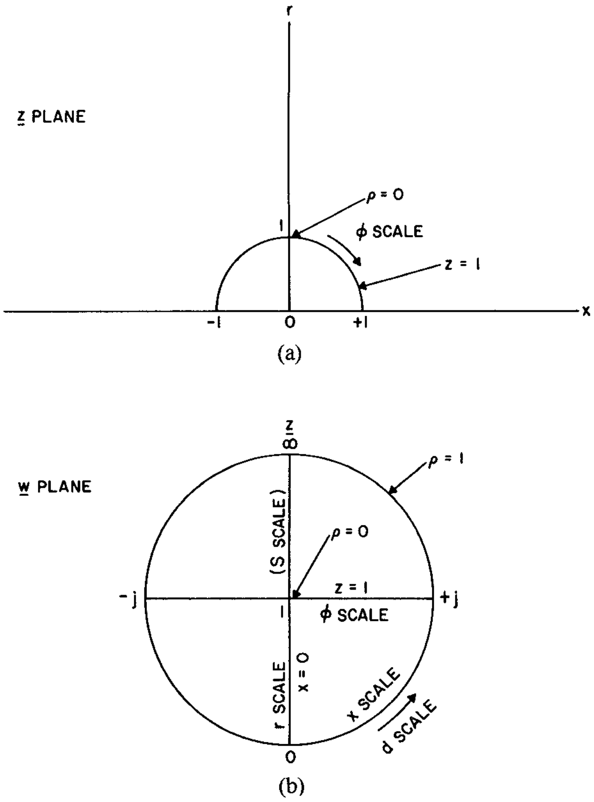


Fig. 6. The two coordinate systems used for impedance charts. (a) The plane of impedance ratio (polar or rectangular coordinates). (b) The plane of reflection ratio (hemisphere chart of impedance ratio).

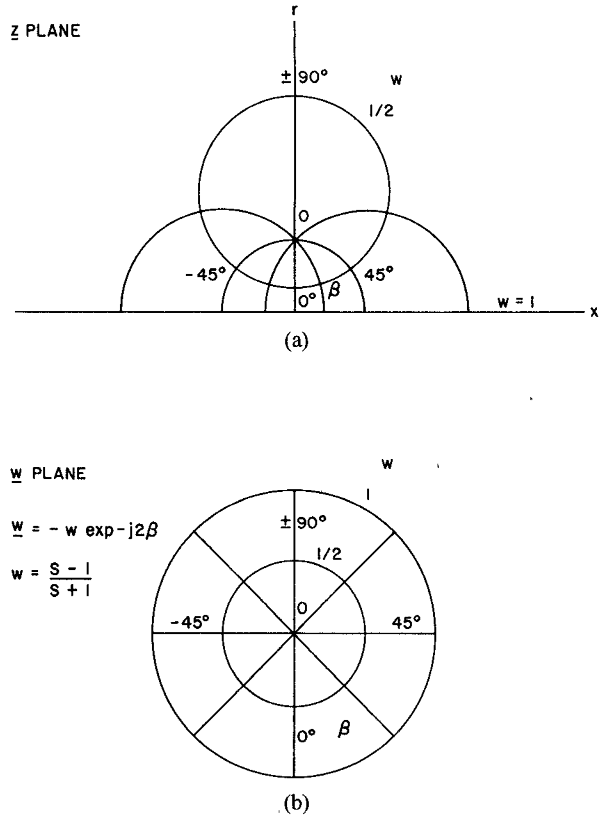


Fig. 7. The polar components of the reflection ratio.

charts [1], [2]. That should have served as a precedent for later practices.

Fig. 5 shows the adaptation of such coordinates to the hemisphere chart, which is the circle of the reflection coefficient. Fig. 5(a) shows the conventional coordinates used by Carter [5]. Fig. 5(b) shows the pattern of positive-real impedance on these coordinates, used by Carter and later by Smith in his book [19]. Fig. 5(c) shows the upside-down pattern first used by Smith but later superseded by Fig. 5(b). Fig. 5(d) shows the pattern of positive-real impedance on the logical coordinates which I adopted when I began to use the hemisphere chart. Unfortunately, the conventional display in Fig. 5(b) is most commonly used for the familiar Smith Chart. Both have the same concentric circles of the reflection coefficient, VSWR, reflection loss, etc. Peculiarities of an impedance locus are similarly visible in both forms.

VI. Relations Between Impedance Ratio and Reflection Ratio

The impedance ratio was previously plotted on rectangular coordinates ($z = r + jx$) to display a locus over a range of frequency or some other parameter. Here, the same locus is plotted on polar coordinates of the reflection ratio (reflection coefficient, $w = u + jv$) within a unit circle. The transformation of this locus from either plane to the other is determined by the relation between the two sets of coordinates. These relations are shown in simple form in Figs. 6–10.

Fig. 6 shows the transformation from the positive-real half-plane of impedance to the inside of a circle of reflection ratio. These are the two projections of one hemisphere as shown in Fig. 3.

Fig. 7 shows the same coordinates with typical loci of the reflection ratio in polar coordinates. In Fig. 7(a), the complete circle of constant reflection (w or ρ) on the z plane is the same as previously used by Smith [1] and the author [2], [3]. In those cases, we were not concerned with the angle of reflection, which is shown by an orthogonal set of circles. Fig. 7(b) shows merely the polar coordinates of the reflection ratio.

Fig. 8 shows the transformation from the impedance ratio in polar coordinates (a) to latitude and longitude on the hemisphere chart (b). The latter is the simplest map of the impedance ratio on the hemisphere chart. It was shown by Carter [5] so it is designated the Carter Chart. This is the form I adopted in 1942 for use as graph paper. It has the simplest rules for inversion.

Fig. 9 shows the transformation of the rectangular coordinates of impedance to a set of orthogonal circles on the hemisphere chart. The two coordinates are shown separately on the two halves for clarity. Fig. 10 shows the same for the inverse impedance ratio, which is the admittance ratio. Figs. 9(b) and 10(b) are the Smith Chart [6], [10], [19]. They are suited for adding series components of impedance, then inverting and adding parallel components of admittance. The Smith Chart is most widely used, and has been found to be extremely useful.

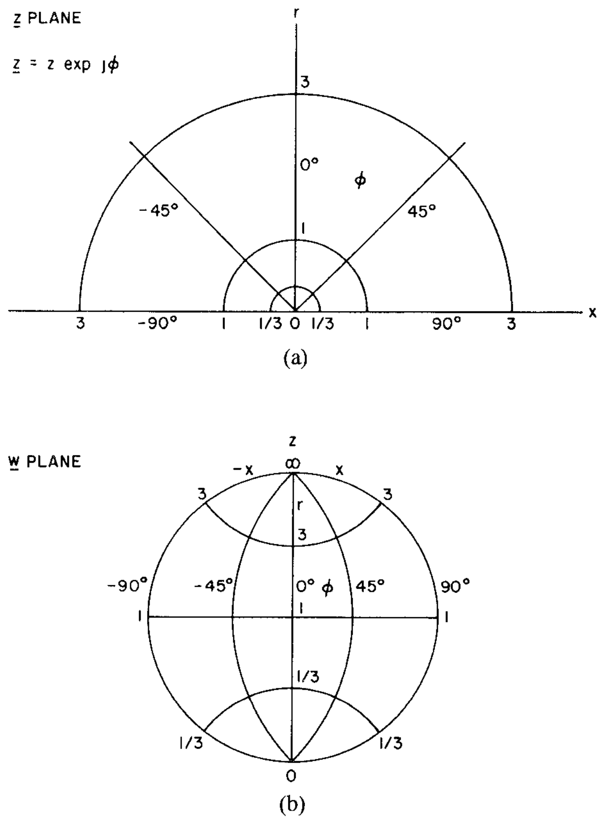


Fig. 8. The polar components of the impedance ratio.

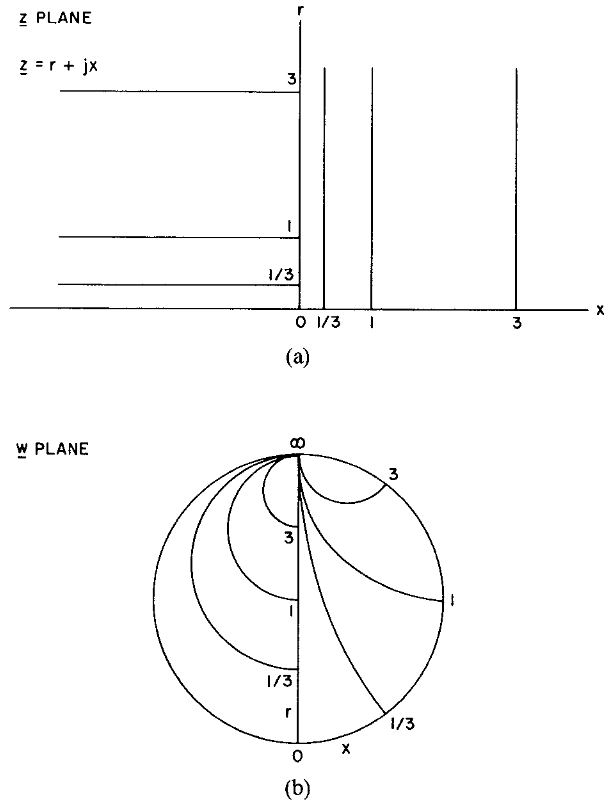


Fig. 9. The series rectangular components of the impedance ratio.

VII. Transmission-Line Transformation of Impedance

If a length of line is inserted between the driving point and an impedance, the impedance is transformed to present a different value. If the line impedance (Z_0) is taken as the reference on a hemisphere chart, any point on the impedance locus is modified in two ways.

- It is rotated by an angle equal to double the phase length of the line.
- Its radius is decreased by the power ratio of attenuation (if any) in the line.

Each of these results reflects the round-trip effect of the line on the reflection coefficient of the load impedance. Their simplicity is a remarkable peculiarity of the hemisphere chart.

Referring to Fig. 11, any point on the chart may be regarded as the impedance of a length of line on short circuit. The reflection coefficient is reduced from unity and retarded in angle by the round-trip attenuation and phase $(2\alpha + j2\beta)$. The complex reflection coefficient becomes

$$w = -w \exp - j2\beta = -\exp -2\alpha \exp -2\beta$$
$$= -\exp -2(\alpha + j\beta). \tag{1}$$

The minus sign is required for zero impedance of zero length on short circuit.

In general, any complex load impedance can be regarded as a length of line on short circuit (α', β'). Then, an inserted length may add its values (α'', β''). The resulting impedance is that of the sum (α, β). At any stage, the impedance ratio is

$$z = \frac{1+w}{1-w}. \tag{2}$$

There are a few familiar rules for impedance transformation by a length of lossless line at one frequency.

- A quarter-wave line ($\beta = \pi/2$, $d = \lambda/4$) inverts the impedance on the chart. For example, a low resistance is multiplied by S^2.
- A half-wave line ($\beta = \pi$, $d = \lambda/2$) rotates by one circle and therefore leaves the impedance unchanged.

VIII. Transmission-Line Measurement of Impedance

There are two methods commonly used for measurement of an impedance connected to the far end of a lossless transmission line. The near end is connected with a signal generator of the desired frequency range. The methods are diagramed in Fig. 12.

The original method is the one proposed by Carter [5]. A sliding probe is weakly coupled with a long line to observe minimum and maximum voltage (or current) and the location of the minimum (being sharper than the maximum). From these observations

$$S = \frac{\max V}{\min V}; \quad \rho = \frac{S-1}{S+1} = w; \quad \beta = 2\pi d/\lambda \text{ or } (360°)d/\lambda \tag{3}$$

$$w = -w \exp j2\beta; \quad z = \frac{1+w}{1-w} = Z/Z_0. \tag{4}$$

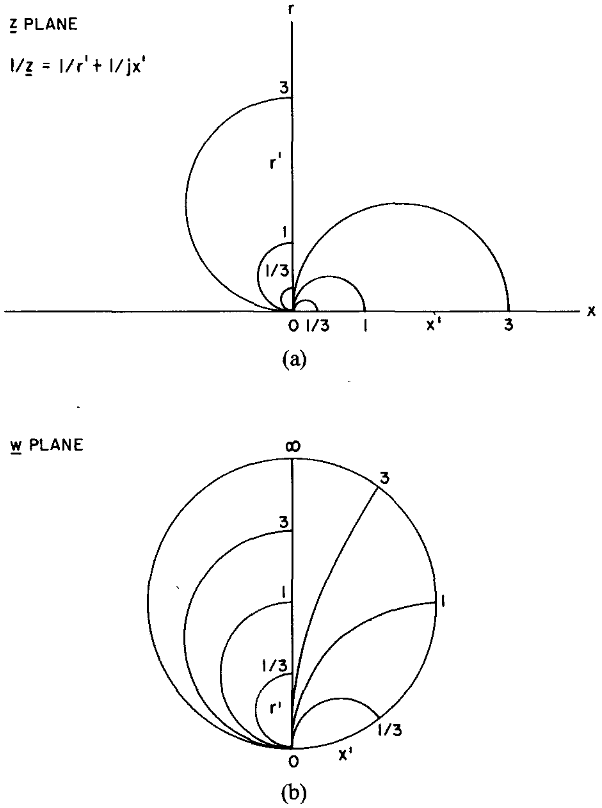


Fig. 10. The parallel rectangular components of the impedance ratio.

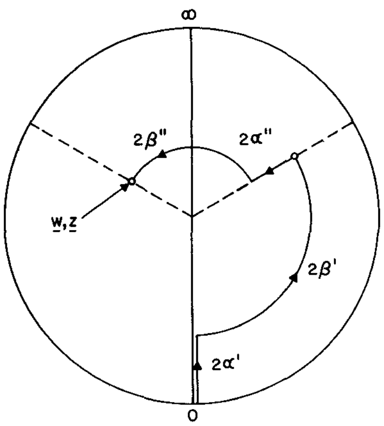


Fig. 11. Sum of length quantities around the chart.

In effect, the impedance at min V is measured to be Z_0/S. Then, the load impedance is computed by subtracting the intervening length of line ($d = (\beta/2\pi)\lambda$).

The later method evolved with the development of the directional coupler. A weak coupler at a point on the line enables direct measurement of the complex reflection ratio, as indicated. The distance to the load (d_c) is subtracted as above to give the reflection coefficient at the load, which determines the complex load impedance.

This method is used in the Hewlett-Packard line of the most advanced impedance measuring equipment [25]. Its utility is restricted to a high-RF range (typically above 1 MHz). It happens that one type of coupler has directivity nominally independent of frequency. That is the parallel-line coupler invented long before by Affel at BTL and

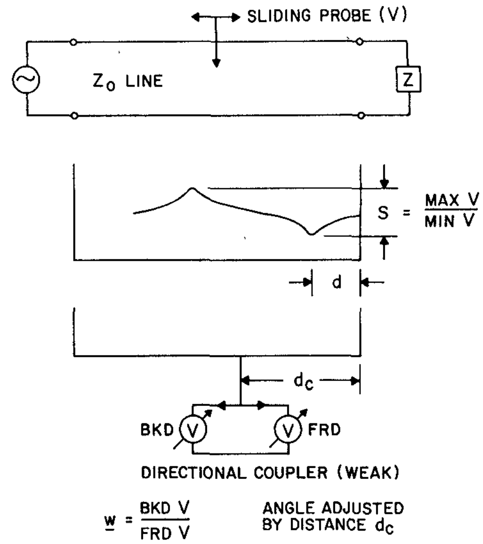


Fig. 12. Two methods for measurement of impedance with reference to a transmission line.

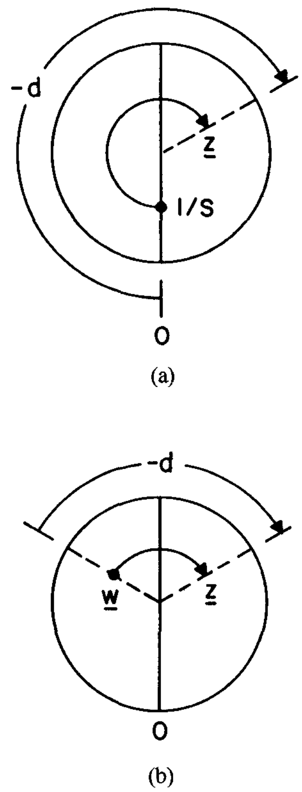


Fig. 13. Diagrams of the two methods of impedance measurement. (a) By sliding probe on slotted line. (b) By directional coupler.

Pistolkors in Russia. I adapted it to the coaxial line about 1944 [11]. The usual value of Z_0 is 50 Ω.

In Fig. 13, these two methods are explained with reference to the hemisphere chart.

Fig. 13(a) shows the method of a sliding probe, usually on a coaxial line. A common line is a round inner conductor between two planes. It was used by the author from 1944, and later was adopted by Hewlett-Packard. The VSWR (S) is measured and the location of minimum impedance (resistance ratio $=1/S$) is noted (d). It is taken as the zero reference in Fig. 13(a). From the point $1/S$ on the zero axis, the radius is rotated backward by d to locate the impedance ratio (z) being measured.

Fig. 13(b) shows the method of a directional coupler, usually a backward coupler in a coaxial line. First, the effective length of line between coupler and load is usually determined for short-circuit or open-circuit termination. It is determined by the angle of the (unit) reflection coefficient observed on the coupler. Then, the impedance to be measured is connected, the reflection coefficient (w) is observed in terms of magnitude and angle, and it is located on the chart. The radius is rotated backward by the known length of line to locate the impedance ratio (z) being measured.

For either method, the chart gives a graphical view of the process.

IX. The Use of a Blank Chart

Some of the principal uses of the hemisphere chart do not require a complete grid of coordinates:

- to display the behavior and critical frequencies of an impedance locus over a frequency range;
- to show its compliance with tolerance circles of reflection (commonly VSWR in decibels);
- to take automatic mechanical plotting of a locus by frequency sweeping.

Then, an overlay or underlay may be used to provide any desired set of coordinates (and at any orientation). This enables the plot to be interpreted or read in terms of such coordinates.

For these uses, we have come to prefer a blank chart for avoiding the confusion of a "busy" grid of coordinates (notably the Smith Chart). Fig. 14 shows a skeleton chart for this purpose. It contains the essential scales, from which any interpretation can be derived by some simple construction. A blank chart the same size as the available Smith Charts can be made with the skeleton scales. Then one may select any one of the available grids for use as overlay or underlay. This retains the convenience of the skeleton chart for annotations. We adopted this practice at Wheeler Laboratories in 1948. It superseded the Carter Chart, which had been used as graph paper by some of the same group previously in the Hazeltine Little Neck Laboratory. The same practice is continuing in the successor group now active in the Research Laboratories of Hazeltine Corporation.

X. Examples of Use

The number and variety of uses of the hemisphere chart are found in the extensive literature on the subject. The most prolific source may be the IEEE Transactions on Microwave Theory and Techniques. Smith's book is one collection [19]. Two typical examples are given here. As usual, the locus is a trace of impedance variation over a range of frequency. The frequency scale has to be marked on the locus.

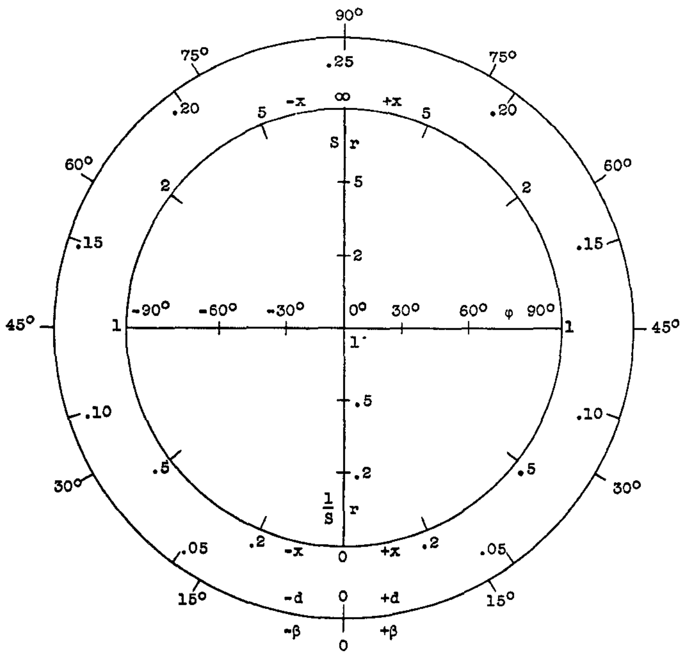


Fig. 14. The scales on the skeleton hemisphere chart.

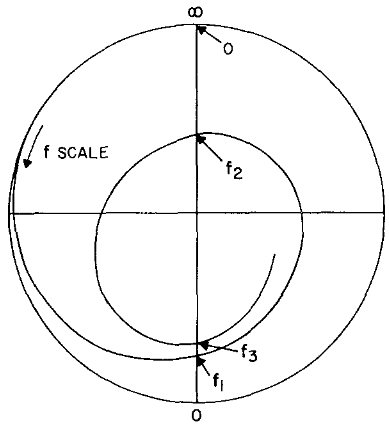


Fig. 15. Frequencies of resonance in a long biconical dipole antenna.

Because the hemisphere chart had its origin as a chart of reflection at the end of a transmission line, its principal applications have been related to that subject. Various other applications are a by-product of its properties. The most common use of such a line has been the connection of radio equipment with a remote antenna. Therefore, a typical use of the reflection chart is the presentation of the frequency locus of the complex impedance of an antenna.

Fig. 15 shows the impedance locus of a long, rather thin, biconical antenna. (The biconical is chosen, rather than a cylindrical wire, to avoid excess capacitance at the terminals.) Its half-wave first resonance is at f_1 and its higher resonances are at f_2, f_3, etc., near integral multiples of f_1. The following are notable features of the locus on the reflection chart.

- Each frequency of resonance appears near a minimum or maximum of impedance.

- Each frequency of resonance may be taken at the point of zero angle, which is the crossing of the vertical axis, as indicated.
- The locus on the hemisphere chart is a spiral tending toward convergence at higher frequencies.
- The rotation is proportional to the length in wavelengths.
- The convergence is proportional to the radiation of power from a traveling wave along the length.
- At frequencies much below the first resonance, the locus approaches the rim of the chart, which indicates the small radiation power factor of a "small antenna" [23].
- At higher frequencies, the spiral converges toward a point representing the wave resistance of the biconical conductor (here taken as a reference at the center of the chart).

Most of these features are characteristic of any reflection chart.

Fig. 16 shows another application. It is related to an antenna or other load with rather narrow resonance at one frequency (f_0). A common objective is a close approximation to matching the load impedance to a line over a frequency band (f_1 to f_2) wider than the resonance. The deficiency of matching is described by the tolerance of reflection within the bandwidth (in terms of $\rho > 0$ or $S > 1$). There are two parts to the matching process:

- transforming the load impedance to fall within the smallest circle centered on the axis of pure R;
- transforming this center to the line wave resistance.

The most common and most elementary form of wide-band matching is "double tuning" [24]. It was first used by the author in 1942, and in various other laboratories around that time. It is accomplished by providing in the matching network a second resonance at a frequency near f_0, between f_1 and f_2. With reference to Fig. 16, it is here developed on the hemisphere chart.

- Z_a is the locus of the load, taken to be an antenna of the type represented in Fig. 15. It has its lowest resonance at f_0. It is a series resonance at minimum impedance. At edgeband (f_1, f_2), it has a greater impedance, which is here taken as the reference equator on the chart. Then, the locus is similar to a contour of constant R and varying X. The ends of the Z_a locus give greatest reflection, which is to be reduced by the matching network.
- The ends of Z_a are on a contour of constant conductance (constant parallel R) shown by a dashed line. A parallel-resonant network is designed to add opposite shunt susceptance. This brings together the ends of the locus and thereby minimizes the tolerance circle around the locus of the resulting Z_b.

This simple result could be derived by analysis, but the chart gives a clear picture of the relations. It is easy to prove on the chart:

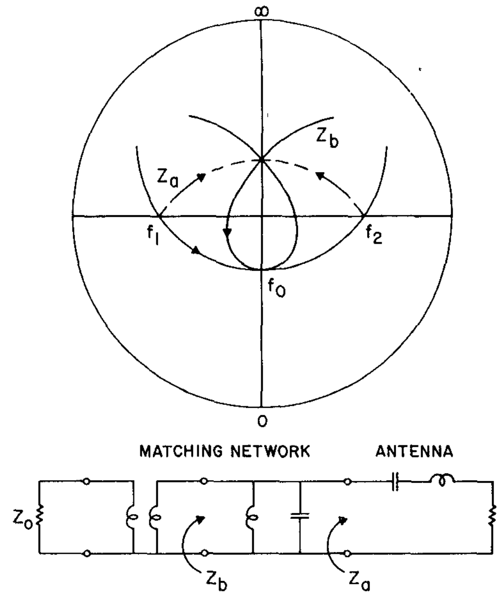


Fig. 16. Synthesis of double tuning for wide-band matching.

- that this construction gives the smallest tolerance circle possible by this network;
- that the reflection coefficient is squared by this process ($\rho_b = \rho_a^2$).

Having achieved this result centered on the special reference equator, the center can be transformed to the line impedance (Z_0) by a simple transformer, as shown.

XI. The Logarithmic Impedance Chart

About 1948 [15], I used another set of coordinates which were also suited for a reflection chart with vertical axis of real impedance. It is a logarithmic grid with compatible scales of magnitude and angle of impedance. It offers the feature that the shape of a locus is independent of the reference or the level of impedance.

The feature of this chart is the compatible scales of magnitude and angle. There is a set of scales that will give a circular contour of reflection coefficient in the limit of a small circle. Equal distance on both scales should correspond to the pair of values on either row here:

Magnitude	*Angle*	*Angle bounds*
one napier	one radian	$\pm\pi/2$
0.6822 decade	90 degrees	± 90

Fig. 17 shows this chart as it appeared in my 1948 monograph [15]. Compatible scales gave equal weight to vertical and horizontal distance. There was no need for contours of the reflection coefficient. The motivation was to place the ratio mean at the middle of a line joining any two points of impedance. This is exemplified by the sloping dashed line.

This chart was a half-solution of the problem of crowding toward the rim of the hemisphere chart. It removes the

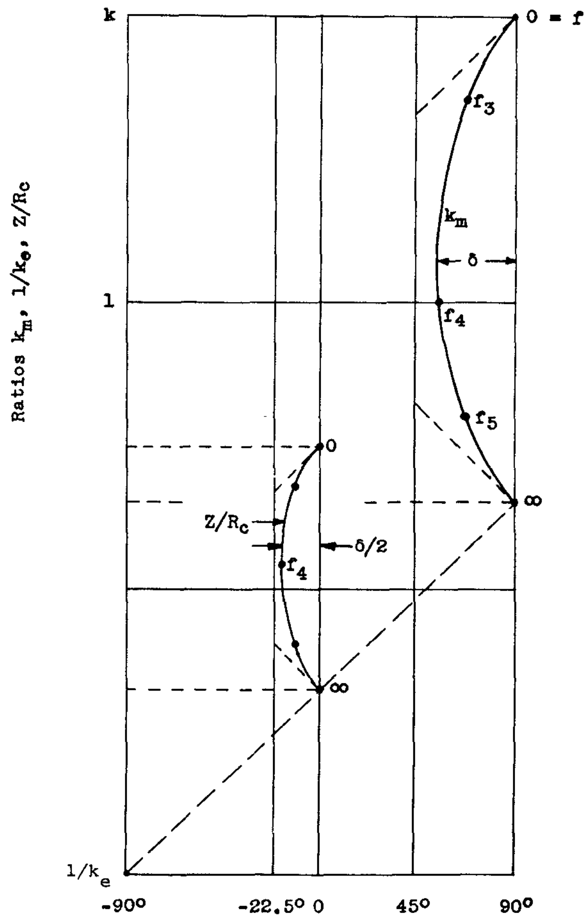


Fig. 17. The logarithmic impedance chart.

top and bottom bounds by the log scale. However, it leaves the side bounds at one quadrant of angle, and therefore the crowding toward these bounds.

XII. The Logarithmic Reflection Chart

A few years ago, I again faced the two problems of the hemisphere chart, namely,

- the shape of a locus depending on a reference value, and
- the compression of a locus toward the rim of the chart.

An angle scale was needed to go with the log scale of magnitude. I perceived a unique basis for a log scale of angle:

- the vertical log scale of impedance is also a log scale of VSWR with zero angle;
- a horizontal log scale of the VSWR caused by angle with level magnitude.

I name these scales $\log S_z$ and $\log S_x$. They could be plotted on log–log coordinates in terms of S_z or Z (vertical) and S_x (horizontal).

The following additional symbols are used in this section:

$z = \ln Z = z_0 + \ln S_z =$ vertical scale.

$x = \ln S_x =$ horizontal scale.

$S_z = \exp(z - z_0) = Z/Z_0 = S^{\pm 1}$ on z axis.

$S_x = \exp x = S^{\pm 1}$ on $\pm x$ axis.

$q = X/R = \tan\phi = \sinh x$.

(Distinguish from Q for X/S of a single reactor, as in a resonant circuit.)

Note relating to the functions ln, exp, sinh, cosh:

- the natural functions (base e, napiers) are used for convenience in theoretical relations;
- the corresponding decimal functions (base 10, decades) can be used for compatibility with the log scales on graph paper.

By way of introduction, Fig. 8(b) shows the Carter Hemisphere Chart [5]. Like the Smith Chart [6], [10], the basic map of polar coordinates is the reflection-coefficient magnitude and angle on a circular area. The Carter Chart has superposed impedance loci in terms of Z and ϕ. Each of these families is made of orthogonal circular arcs. The ϕ loci converge toward the poles. Extreme values are crowded toward the rim of the chart. A reference (Z_0) must be assigned, and then it determines the shape of any impedance locus that may be traced over a range of frequency. The chart to be described is based on rectangular logarithmic coordinates of Z and a function of ϕ.

The logarithmic reflection chart [21], [23] is a departure from the hemisphere chart in two respects.

- The vertical scale is a logarithmic scale of impedance magnitude (Z) so the shape of a locus is independent of the reference value (Z_0).
- The horizontal scale is a logarithmic scale related to the impedance angle (ϕ) in such a way as to give a like variation with reflection coefficient.

It follows that a locus of reflection coefficient has equal diameters on both scales. It departs from a circle by equal flattening in all quadrants.

Referring to Fig. 18, the vertical scale is a log scale of impedance magnitude (Z). On the vertical axis, the same log scale gives the VSWR relative to a reference value (Z_0). This VSWR is here denoted

$$S_z = Z/Z_0. \tag{5}$$

The log scale becomes

$$z = \ln Z = \text{lh}\, Z_0 + \ln S_z. \tag{6}$$

Here, S_z may be greater or less than unity so z may be positive or negative.

At the reference level of Z_0, an angle of impedance (ϕ) would cause a VSWR here denoted

$$S_x = \frac{1+\tan\phi/2}{1-\tan\phi/2} = \sec\phi + \tan\phi = \frac{1}{\sec\phi - \tan\phi}. \tag{7}$$

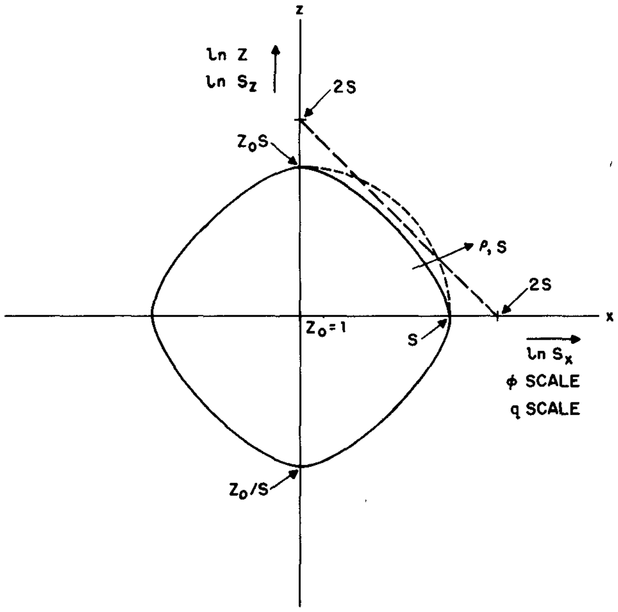


Fig. 18. The basis for the logarithmic reflection chart.

The log scale becomes

$$x = \ln S_x = 2\,\mathrm{atah}\tan\phi/2 = \mathrm{asinh}\tan\phi = \mathrm{acosh}\,(1/\cos\phi) \tag{8}$$

Here, likewise, S_x may be greater or less than unity so x may be positive or negative, corresponding to ϕ.

A locus of the reflection coefficient (magnitude ρ) corresponds to the familiar VSWR:

$$S = \frac{1+\rho}{1-\rho} > 1; \quad \rho = \frac{S-1}{S+1} < 1. \tag{9}$$

The locus intersects the two axes at

$$S_z = S^{\pm 1} \text{ and } S_x = S^{\pm 1}. \tag{10}$$

These points are shown in Fig. 18.

The two scales ((6) and (8)) are taken to evaluate, respectively, Z and ϕ at any point on the x, z plane. The corresponding value of the reflection coefficient (in terms of ρ or S) is thereby determined over this plane, so a locus can be graphed for any constant value. An example of such a locus is shown in Fig. 18, centered on $Z_0 = 1$. It has equal diameters on the crossed axes

$$2z \text{ or } 2x = \ln S - \ln 1/S = 2\ln S. \tag{11}$$

It departs from a circle by some degree of flattening in each quadrant. The vertical log scale assures that the shape of this locus would be the same for a center at any reference value Z_0 on the vertical axis.

The most significant feature of the log chart is the reflection locus, compared with the familiar circle centered on the hemisphere chart. Referring to the typical locus shown in Fig. 18, these points may be noted.

- Like a circle, it has equal crossed diameters and four quadrants of like shape.
- Departing from a circle, each quadrant is flattened in some degree.

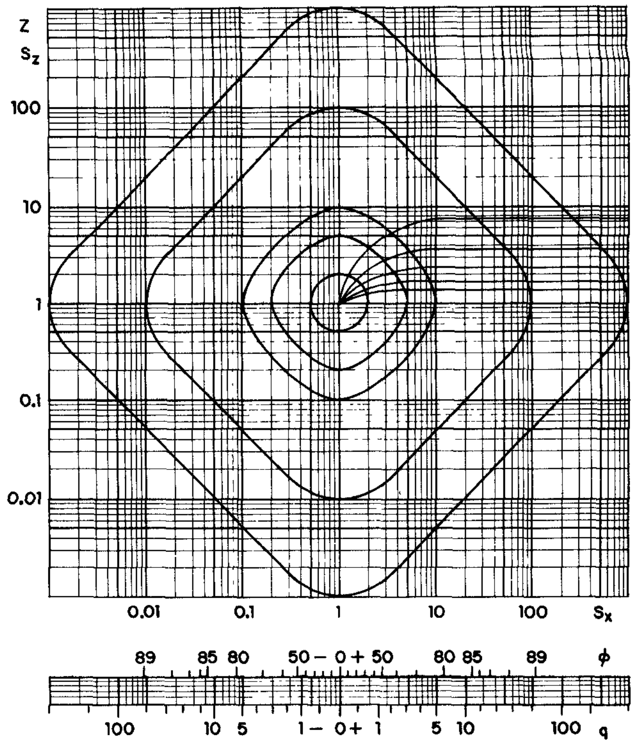


Fig. 19. Reflection contours on the chart.

- The size is unbounded, thereby avoiding the crowding near the rim of the hemisphere chart.
- The size and shape are the same for a center at any reference Z_0 on the vertical axis.

The departure from a circle is a disadvantage, while the last two features are the advantages that make the log chart especially useful for some purposes.

The size of a locus increases with the log of VSWR, so it has no bounds, and a wide range can be presented in a moderate space. Decimal log scales are most convenient, and the usual log–log graph paper is suitable.

Fig. 19 shows a decimal log chart with a family of reflection loci. The two scales are:

- vertical Z or $S_z R_0$;
- horizontal S_x.

On the horizontal scale, from (7):

$$\phi = 2\,\mathrm{atan}\frac{S_x-1}{S_x+1} = \mathrm{atan}\,\frac{1}{2}(S_x - 1/S_x) \tag{12}$$

$$R = Z\cos\phi; \quad X = Z\sin\phi. \tag{13}$$

These relations describe the impedance at any point on the chart.

The locus of any reflection coefficient (ρ) can be shown to be

$$\frac{1+\rho^2}{1-\rho^2} = \cosh x \cosh z = \frac{1}{4}(S_x + 1/S_x)(S_z + 1/S_z). \tag{14}$$

It is easily evaluated in terms of the natural log (x, z) and

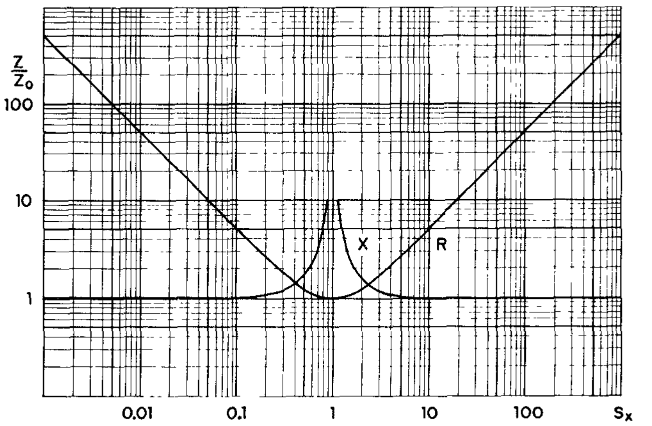


Fig. 20. Impedance contours on the chart.

converted to decimal log. This form shows the symmetry on both axes.

The locus approaches a small circle near the center:

$$(2\rho)^2 = x^2 + z^2. \qquad (15)$$

Far out, the locus approaches a large diagonal square:

$$2S = S_x S_z. \qquad (16)$$

The circle and square are outer bounds, as seen in Fig. 18. They are tangent if $S = 5.33$; then the 45° radius of the locus is less than that of the outer bounds by the factor 0.925. This factor is an example of the flattening of the arc in each quadrant.

For computations, it is helpful to express the angle scale in terms of $q = X/R$, which retains the sign of the angle:

$$q = X/R = \tan\phi = \frac{1}{2}(S_x - 1/S_x) = \sinh x \qquad (17)$$

$$s_x = \sqrt{1+q^2} + q = \frac{1}{\sqrt{1+q^2} - q}. \qquad (18)$$

Below the log–log grid in Fig. 19, there are supplemental horizontal scales in terms of the angle (ϕ) and its tangent (q). They are converted to S_x by (7) and (18).

For graphical presentation of impedance on any scale of frequency, it has been customary to use a log scale of Z and R. Both of these are positive. There has been no logical logarithmic presentation of X or ϕ, both sides of zero. The S_x scale offers a unique solution to this problem. It represents $\pm\phi$ above and below unity on the log scale. If the Z variation is plotted from a reference (as $S_z = Z/Z_0$), the S_x graph may be superimposed about the same unity level on the log scale.

A pair of R and X loci have the shape shown in Fig. 20. The pair is based on one value of Z_0. A family is formed by vertical displacement with the same shape, a feature of the log chart.

$$R \text{ locus: } Z/Z_0 = \exp(z - z_0) = R\cosh x = R/\cos\phi$$
$$= R\left(1 + \frac{(S_x - 1)^2}{2S_x}\right). \qquad (19)$$

$$X \text{ locus: } Z/Z_0 = \exp(z - z_0) = X/\tanh x = X/\sin\phi$$
$$= X\left(1 + \frac{2}{S_x^2 - 1}\right). \qquad (20)$$

A particular set of coordinates has been devised for presentation of any impedance locus. The scales are logarithmic in terms of VSWR, which is a measure of the reflection coefficient. A locus of constant reflection has equal diameters on the crossed axes but differs from a circle. The shape of a locus is independent of the choice of a reference value of impedance.

This logarithmic reflection chart has been found especially helpful in presenting the wide range of impedance, which is typical of a small antenna and a wide-band matching network for such an antenna.

XIII. Conclusion

A chart of contours of the reflection coefficient has evolved through several principal forms. Each has one of these distinctive grids of coordinates:

1) rectangular coordinates of the complex impedance;
2) polar coordinates of the reflection coefficient (the hemisphere chart);
3) rectangular coordinates of the log of impedance (compatible scales of magnitude and angle);
4) rectangular coordinates of the log of impedance magnitude and a compatible function of the angle (the logarithmic reflection chart).

After the first, each form offers a particular grid on which may be plotted a frequency locus of impedance. The second is the one in most common use today, as the Smith Chart or the Carter Chart. The last offers an opportunity for relief from some limitations of the hemisphere chart, but at the expense of some other valuable features.

The subject of these charts is a branch of the technology of graphic presentation of relations among some variables in a physical system. In this case, the variables are frequency and impedance. The reflection at the end of a line is an intermediate quantity which is particularly relevant for some applications. A chart of the reflection coefficient has proved helpful in various ways in the course of its evolution in different forms.

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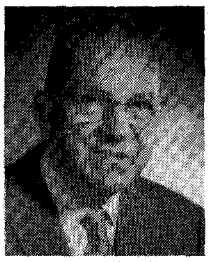

Harold A. Wheeler (A'27–M'28–F'35–LF'68) was born in St. Paul, MN, on May 10, 1903. He received the B.S. degree in physics in 1925 from George Washington University, Washington, DC, and in 1972 the honorary degree of Doctor of Science. He continued post-graduate studies until 1928 at Johns Hopkins University, Baltimore, MD. In 1978, he received from Stevens Institute of Technology the honorary degree of Doctor of Engineering.

He was employed by the Hazeltine Corporation from 1924 to 1946, advancing to Vice-President and Chief Consulting Engineer. In 1959, he resumed activity with this company as a Director, and is now Chairman Emeritus and Chief Scientist. From 1947, he was President of Wheeler Laboratories, Inc., Great Neck, NY, which became a subsidiary of Hazeltine Corporation, and in 1971 merged into the parent company.

His activity in the field of microwaves dates back to World War II, when he was one of the leaders in the Combined Research Group at NRL. That group was developing the future system of IFF (Interrogation Friend-or-Foe), then designated the Mark V. From that beginning grew the Mark XII, which is now the standard. In the Wheeler Laboratories, during the two decades after the war, he directed advanced work on microwave antennas and circuits, largely for precision tracking radar.

Mr. Wheeler has served the IRE in such positions as Director (1934, 1940–1945) and Chairman of the Standards Committee; he has contributed many papers to IRE periodicals and received the Morris N. Liebmann Memorial Prize from IRE in 1940. In 1964, he was awarded the Medal of Honor by IEEE and the Armstrong Medal by the Radio Club of America. In 1975, he was the second to receive from MTT-S the Microwave Career Award. He has been granted 180 U. S. Patents and many foreign patents. He is a Fellow of the Radio Club of America, and Associate Fellow of AIAA, a Member of IEE (British), and a member of Sigma Xi and Tau Beta Pi.